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**“New Mathematical Methodology for Stochastic Control  
Problems And Its Application to Risk Management”**

Facing the risk of market uncertainty, (ir)reversible investment problem arises naturally in resource extraction and power generation: companies extract resources (such as oil or gas) and choose the level of installed capital in response to the random fluctuation of the market price for the resource, subject to certain capital constraints, as well as the associated costs of capacity expansion and reduction: the costs of decreasing and increasing capital (where a negative cost represents a partial recovery of the initial investment), the running cost of installed capital, and the cumulative cost. The goal of the company is to maximize its long-term profit, subject to these constraints and the rate of resource extraction.

Arising first in the economics literature, this kind of risk management problem has attracted the interest of both the applied mathematics and economics communities. Often formulated as a continuous time multi-dimensional control problem, it is closely related to optimal portfolio/consumption management problem in financial markets. The mathematical analysis has evolved considerably from the initial heuristics to the more sophisticated and standard stochastic control approach; see Bather and Chernoff (1967); Benes, Shepp, and Witsenhausen (1980); Karatzas (1983), Harrison and Taksar (1983), Davis, Dempster, Sethi, and Vermes (1987); Kobila (1993); Brekke and Åksendal (1994); Abel and Eberly (1997); Baldursson and Karatzas (1997); Åksendal (2000); Scheinkman and Zariphopoulou (2001); Wang (2003); Chiarolla and Haussmann (2005); Bank (2005); Guo and Pham (2005); Merhi and Zervos (2006); Alvarez (2006); and a standard reference on irreversible investment, Dixit and Pindyck (1994).

However, almost all of this work assumes a special diffusion process such as geometric Brownian motion for the price process and deals with payoff functions which are continuously differentiable, monotonically increasing, and strictly concave. These assumptions are not satisfied for most (ir)reversible investment problems, for which there are no regularity properties for either the value function or the boundaries. Moreover, boundaries between the action and no-action regions are not necessarily monotonic. Therefore, the traditional approach could lead to misleading results.

To address these issues, we propose to study 1) the necessary and sufficient conditions for the regularity properties; and 2) the characterizations of value functions and of the action and no-action regions when these regularity conditions fail. We stress that proper understanding of these issues is extremely important when explicit solutions are unavailable and numerical solutions are necessary. Finally, we propose to 3) develop a new computational method based on the theoretical framework.